

Diffraction-corrected neutrino flux and νN total cross section

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Abstract

A neutrino produced in a particle decay maintains an unusual wave nature that reveals a diffraction phenomenon in a large spatial region. Diffraction gives a new component to a neutrino flux in this region and modifies determinations of physical quantities in neutrino experiments. A total cross section of neutrino nucleon scattering at high energies is studied in this paper. A cross section is proportional to an energy in the quark-parton model and is modified with the energy-dependent neutrino flux induced by the diffraction component. A total number of events divided by the energy is not constant but has a slight energy dependence. Theoretical values agree well with the recent experiments.

Keywords: Neutrino diffraction, Neutrino flux

1. Neutrino diffraction and absolute neutrino flux

In high energy neutrino experiments, neutrinos from pion decays are used. Neutrino flux is not directly measurable and is estimated from a pion's flux and decay probabilities. Now precision measurements are becoming possible in neutrino experiments and a precise value of the flux is required. So it is important to verify a standard assumption and to find a precise value. We point out that a correction due to a quantum mechanical interference of a single neutrino is necessary in the flux in near detector regions.

Decay amplitudes and probabilities are obtained normally with a standard method of S-matrix using plane waves. An asymptotic boundary condition [1, 2] is assumed and an average decay rate is found and used for estimations of the flux. This method gives precise values of fluxes of final states in the asymptotic space-time region where a particle has no correlation and is described with a plane wave. It has never been clarified, however, if particles in the final state are observed in an asymptotic region. It would be reasonable to assume that a boundary between an asymptotic and non-asymptotic regions is of microscopic size, since transitions of weak decays occur in short distance regions. However we find that is not the case. An asymptotic region for a neutrino in a particle decay is located in a macroscopically distant area from an initial particle's position. Waves produced at different positions accumulate at a detector and make experiments in the inside of this area unusual. Superposition of waves causes an

interference effect in a form of a diffraction in a parallel direction to the momentum, which makes the plain waves non-asymptotic. Various relations satisfied for the asymptotic states, such as a unitarity, an energy momentum conservation, and others are modified with the diffraction term.

To study the interference phenomenon of particles produced in a decay, it is convenient to study a coherence length from a behavior of a correlation function defined with a wave function. Let $\psi(x, \gamma)$ be a wave function of a final state composed of one particular particle at a space-time position x and others, γ . An amplitude of observing this particle at a momentum p and a position X using a detector is defined with the wave function at one space-time position x and a wave packet $w(p, x)$ in the form

$$\Psi(X, p, x, \gamma) = w(p, x - X)\psi(x, \gamma), \quad (1)$$

where $w(p, x - X)$ is a wave packet with which this particle interacts and a Gaussian form with a size σ and velocity $\vec{v} = \vec{p}/E$, $E = \sqrt{\vec{p}^2 + m^2}$ is used for the sake of simplicity,

$$w(p, x - X) = N_0 e^{-\frac{(\vec{x} - \vec{X} - \vec{v}(x^0 - X^0))^2}{2\sigma}} - ip \cdot (x - X), \quad (2)$$

where $p \cdot x$ is scalar product of a four dimensional coordinate x and momentum p and N_0 is a normalization factor.

Since the wave packet vanishes at $|\vec{x}| \rightarrow \infty$, it satisfies the asymptotic boundary condition. So the wave

packet is appropriate to study a finite size correction. An integral of the product $\Psi^*(X, p, x_1, \gamma)\Psi(X, p, x_2, \gamma)$ is the probability $C(X, p)$,

$$C(X, p) = \int d^4x_1 d\gamma d^4x_2 \Psi(X, p; x_1, \gamma)^* \Psi(X, p; x_2, \gamma),$$

where $d\gamma$ stands for the volume element of state γ . $C(X, p)$ is written as

$$\int d^4x_1 d^4x_2 w^*(p, x_1 - X) w(p, x_2 - X) \Delta(x_1, x_2),$$

using the correlation function defined with

$$\Delta(x_1, x_2) = \int d\gamma \psi(x_1, \gamma)^* \psi(x_2, \gamma). \quad (3)$$

The correlation function generally has the singular function

$$\delta(\lambda) \epsilon(x_1^0 - x_2^0), \lambda = (x_1 - x_2)^2, \quad (4)$$

or regular functions $e^{i\tilde{m}\lambda}, e^{-i\tilde{m}\lambda}$ where a finite \tilde{m} is determined from dynamics of the system. If one momentum that is conjugate to $x_1 - x_2$ is integrated and the singular function $\delta(\lambda)$ is derived, this gives the form

$$C(X, p) = C^{(1)} e^{-\frac{|\vec{X}|}{l_0}}, |\vec{X}| > l_0, \quad (5)$$

where l_0 is given in the form

$$l_0 = \frac{2|\vec{p}|\hbar c}{m^2}, \quad (6)$$

in a high energy region, $|\vec{p}| \gg m$. Other oscillating or decreasing terms give constants to $C(X)$. So l_0 stands for a coherence length of the wave function. The coherence lengths of a pion and an electron of an energy of 1 [GeV] are

$$l_0^{pion} = \frac{2\hbar c}{0.13^2} [\text{GeV}^{-1}] = 2 \times 10^{-14} [\text{m}], \quad (7)$$

$$l_0^{electron} = \frac{2\hbar c}{0.5^2} [\text{GeV}^{-1}] \approx 10^{-10} [\text{m}], \quad (8)$$

and those of other hadrons are shorter than that of a pion. So charged leptons and hadrons have short coherence lengths of microscopic sizes in an ordinary high energy region. Neutrinos are exceptional and lighter than an electron by 10^6 or more and have coherence lengths,

$$l_0^{neutrino} \approx 10^2 - 10^3 [\text{m}], \quad (9)$$

which is a macroscopic size. From Eqs. (8) and (9), the non-asymptotic region is narrow in charged leptons and very wide in neutrinos.

In outside of the coherence length $|\vec{X}| \gg l_0$, the wave-like correlation vanishes. Hence the state behaves like a particle-like object which has no spatial correlation and satisfies the asymptotic condition. In inside, $|\vec{X}| < l_0$, on the other hand, the probability varies with $|\vec{X}|$ and a wave-like correlation remains. So the physical state behaves like an unusual wave that is distinct from a particle-like object. The physical state is not treated as a real particle and does not satisfy the asymptotic condition. So the coherence length is used as the boundary between the asymptotic and non-asymptotic regions.

Transition probabilities in the asymptotic region are studied using an ordinary S-matrix and a decay amplitude of plane waves and hold relations of a standard S-matrix. But those in the non-asymptotic region are peculiar and different from those of particle natures of a standard S-matrix. They violate some relations and are not studied with a standard S-matrix of plane waves. The physics in both regions can be studied with a time dependent Schrödinger equation or a scattering amplitude of wave packets, which satisfy the asymptotic conditions and have manifest position dependence. We study the physics in the non-asymptotic region using the position dependent amplitudes.

Charged leptons are massive and their measurements are made in their asymptotic regions. So physical quantities are computed with the ordinary S-matrix and agree with values obtained in experiments. Weak decays of mesons are understood well with $V - A$ weak interactions through measurements of the charged leptons. For neutrinos the situation is different. Since the asymptotic region is located in a distant area, neutrinos may be observed in non-asymptotic regions. Then the neutrino reveals wave-like phenomena such as interference or diffraction and has an unusual position dependence. A diffraction term, in fact, emerges and gives a finite contribution to the neutrino flux. A new term which does not satisfy various relations of the standard S-matrix is added to an amplitude in a short distance region. The total neutrino's flux thus obtained deviates from that obtained using the naive decay probability and has an excess. Implications of the diffraction term to the total cross sections [3, 4, 5] are studied.

2. Position dependent probability

A diffraction term [6, 7] is derived from a position-dependent amplitude of a pion decay process. The amplitude is determined in the first order of the $V - A$ weak Hamiltonian H_w in the form, $T = \int d^4x \langle l, \nu | H_w(x) | \pi \rangle$, here a pion is prepared at a time T_π , and a neutrino

is measured with a wave packet of a space-time position (T_ν, \vec{X}_ν) and a charged lepton is un-measured. The neutrino wave packet [8, 9, 10] expresses a target nucleon which the neutrino interacts with. Hence the wave packet is well localized in the coordinate variable. To represent this property, the momentum variables must cover whole momentum region and the Gaussian form is used in this work. A set of wave packets of a σ_ν with a continuous spectrum of momentum and coordinate is complete [8]. Hence neutrinos are described with wave packets of central values of the momentum and coordinate and a width [11, 12, 13, 14, 15, 16, 17]. Other particles are described with the plane waves. They are expressed in the form $|\pi\rangle = |\vec{p}_\pi, T_\pi\rangle$, $|\nu\rangle = |\vec{p}_\nu, T_\nu, \vec{X}_\nu, T_\nu\rangle$. The amplitude T is written with the hadronic $V - A$ current and Dirac spinors in the form

$$T = \int d^4x d\vec{k}_\nu N_1 \langle 0 | J_{V-A}^\mu(x) | \pi \rangle \times \bar{u}(\vec{p}_l) \gamma_\mu (1 - \gamma_5) v(\vec{k}_\nu) e^{ip_l \cdot x + ik_\nu \cdot (x - X_\nu) - \frac{\sigma_\nu}{2} (\vec{k}_\nu - \vec{p}_\nu)^2}, \quad (10)$$

where $N_1 = ig(\sigma_\nu/\pi)^{\frac{3}{4}}(m_l m_\nu/E_l E_\nu)^{\frac{1}{2}}$, and the time t is integrated in the region $T_\pi \leq t$. σ_ν is a neutrino wave packet size and is estimated using a nucleus size. The Gaussian form of the wave packet is used for a sake of simplicity but the most important result is unchanged in general wave packets as was verified in [6].

If the coordinate \vec{x} is integrated in Eq.(10), the delta function $(2\pi)^3 \delta(\vec{p}_\pi - \vec{p}_l - \vec{k}_\nu)$ emerges. So \vec{k}_ν is integrated easily and the Gaussian part becomes $e^{-\frac{\sigma_\nu}{2} (\vec{p}_\pi - \vec{p}_l - \vec{p}_\nu)^2 + iE(\vec{p}_\pi - \vec{p}_l - \vec{p}_\nu)(x^0 - T_\nu) - i(\vec{p}_\pi - \vec{p}_l - \vec{p}_\nu) \cdot \vec{X}_\nu}$. The exponent has two stationary momenta, $\vec{p}_l \approx \vec{p}_\pi - \vec{p}_\nu$ and $\vec{v}(x^0 - T_\nu) - \vec{X}_\nu \approx 0$, $\vec{v} = \frac{\partial E(\vec{p}_\pi - \vec{p}_l - \vec{p}_\nu)}{\partial \vec{p}_l}$ [8]. Hence the probability gets a contribution from a lepton around the former momentum, which is called a normal term, and another contribution from it of broad spectrum around the latter momentum, which is called a diffraction term.

The transition probability to this final state is finite and an order of integrations are interchangeable. So the probability is written, after the spin summations are made, with a correlation function and the neutrino wave function in the form

$$\int \frac{d\vec{p}_l}{(2\pi)^3} \sum_{s_1, s_2} |T|^2 = \frac{N_2}{E_\nu} \int d^4x_1 d^4x_2 e^{-\frac{1}{2\sigma_\nu} \sum_i (\vec{x}_i - \vec{x}_i^0)^2} \Delta_{\pi,l}(\delta x) e^{i\phi(\delta x)}, \quad (11)$$

where $N_2 = g^2(4\pi/\sigma_\nu)^{\frac{3}{2}} V^{-1}$, V is a normalization volume for the initial pion, $\vec{x}_i^0 = \vec{X}_\nu + \vec{v}_\nu(t_i - T_\nu)$, $\delta x =$

$$x_1 - x_2, \phi(\delta x) = p_\nu \cdot \delta x \text{ and}$$

$$\Delta_{\pi,l}(\delta x) = \frac{1}{(2\pi)^3} \int \frac{d\vec{p}_l}{E(\vec{p}_l)} (2p_\pi \cdot p_\nu p_\pi \cdot p_l - m_\pi^2 p_l \cdot p_\nu) \times e^{-i(p_\pi - p_l) \cdot \delta x}. \quad (12)$$

In Eq.(12), the momentum, p_l , is integrated in whole positive energy region.

3. Light-cone singularity

$\Delta_{\pi,l}(\delta x)$ is composed of a light-cone singularity [6, 18] and regular terms. The former is generated from those plane waves that have a same phase and are added constructively. In order to extract a leading singular term in the variable, δx , we write the integral in a four dimensional form with a new variable $q = p_l - p_\pi$ that is conjugate to δx . Then $\Delta_{\pi,l}(\delta x)$ is decomposed into integrals in $0 \leq q^0$ and $-p_\pi^0 \leq q^0 \leq 0$. An integral from $0 \leq q^0$ is written in the form, $\{2(p_\pi \cdot p_\nu) p_\pi \cdot (p_\pi - i\frac{\partial}{\partial \delta x}) - m_\pi^2 (p_\pi - i\frac{\partial}{\partial \delta x}) \cdot p_\nu\} \tilde{I}_1$, where

$$\tilde{I}_1 = \int d^4q \frac{\theta(q^0)}{4\pi^4} \text{Im} \left[\frac{1}{q^2 + 2p_\pi \cdot q + \tilde{m}^2 - i\epsilon} \right] e^{iq \cdot \delta x},$$

and $\tilde{m}^2 = m_\pi^2 - m_l^2$. The integrand of \tilde{I}_1 is expanded in $p_\pi \cdot q$ and the integration leads the light-cone singularity [18], $\delta(\delta x^2)$, and less singular and regular terms which are described with Bessel functions. An integral from the region $-p_\pi^0 \leq q^0 \leq 0$, I_2 , is written with the momentum $\tilde{q} = q + p_\pi$ and has no singularity. Thus the correlation function, $\Delta_{\pi,l}(\delta x)$, is written in the form

$$\Delta_{\pi,l}(\delta x) = 2i \left\{ 2(p_\pi \cdot p_\nu) p_\pi \cdot \left(p_\pi - i\frac{\partial}{\partial \delta x} \right) - m_\pi^2 \left(p_\pi - i\frac{\partial}{\partial \delta x} \right) \cdot p_\nu \right\} \times \left[D_{\tilde{m}} \left(-i\frac{\partial}{\partial \delta x} \right) \left(\frac{\epsilon(\delta t)}{4\pi} \delta(\lambda) + f_{\text{short}} \right) + I_2 \right], \quad (13)$$

where $\lambda = \delta x^2$, $D_{\tilde{m}}(-i\frac{\partial}{\partial \delta x}) = \sum_l (1/l!) (2p_\pi(-i\frac{\partial}{\partial \delta x}) \frac{\partial}{\partial \tilde{m}^2})^l$, $f_{\text{short}} = -\frac{i\tilde{m}^2}{8\pi\xi} \theta(-\lambda) \{N_1(\xi) - i\epsilon(\delta t) J_1(\xi)\} - \frac{i\tilde{m}^2}{4\pi^2\xi} \theta(\lambda) K_1(\xi)$, $\xi = \tilde{m}\sqrt{|\lambda|}$, N_1 , J_1 , and K_1 are Bessel functions. f_{short} has a singularity of the form $1/\lambda$ around $\lambda = 0$ and decrease as $e^{-\tilde{m}\sqrt{|\lambda|}}$ or oscillates as $e^{i\tilde{m}\sqrt{|\lambda|}}$ at a large $|\lambda|$.

The series in Eq.(13) converges when $2p_\pi \cdot p_\nu \leq \tilde{m}^2$ and this expression is valid in this kinematical region.

The singular terms, $\delta(\lambda)$ and others, in Eq.(13) decrease slowly with the distance and give a correlation effect in a wide area. These terms are derived from the integration in $E_\pi \leq E_l$. Because this region is outside of kinematical region of satisfying the energy and momentum conservation $p_\pi = p_\nu + p_l$, its contribution

disappears in the amplitude defined at $T = \infty$. Conversely the singular terms are included only in the physical quantities observed at a finite time interval. So they are not derived from the standard calculations of plane waves with the asymptotic conditions. The latter term, I_2 , on the other hand, comes from the integration region $E_l \leq E_\pi$, which is in the kinematical region of satisfying the energy and momentum conservation. So this determines the quantities at $T = \infty$. This term oscillates or decreases fast in λ with a time scale determined with ordinary microscopic quantities and becomes microscopically short, as most other cases. We will see that the light-cone singularity is combined with the small neutrino mass and gives a finite distance correction of an exceptional scale to the position dependent neutrino flux.

4. Integration of space-time coordinates

Next, Eq. (13) is substituted to Eq. (11) and the coordinates \vec{x}_1 and \vec{x}_2 are integrated. The most singular term, $J_{\delta(\lambda)}$, is from $\frac{\epsilon(\delta t)}{4\pi}\delta(\lambda)$ in $\Delta_{\pi,l}$, which has no scale, and is rewritten using the center coordinate $X^\mu = (x_1^\mu + x_2^\mu)/2$ and the relative coordinate $\vec{r} = \vec{x}_1 - \vec{x}_2$. The center coordinate \vec{X} is integrated easily and $J_{\delta(\lambda)}$ becomes an integral of the relative coordinates (\vec{r}_T, r_l) . Finally we have

$$J_{\delta(\lambda)} = C_{\delta(\lambda)} \epsilon(\delta t) |\delta t|^{-1} e^{i\bar{\phi}_c(\delta t) - \frac{m_\nu^4}{16\sigma_\nu E_\nu} \delta t^2}, \quad (14)$$

where $C_{\delta(\lambda)} = (\sigma_\nu \pi)^{\frac{3}{2}} \sigma_\nu / 2$ and $\bar{\phi}_c(\delta t) = \omega_\nu \delta t$, $\omega_\nu = m_\nu^2 / 2E_\nu$. The phase $\phi(\delta x)$ of Eq. (11) becomes the small phase $\bar{\phi}_c(\delta t)$ of Eq. (14) at the light cone due to a cancellation between the time and space components. The angular velocity of $\bar{\phi}_c(\delta t)$ is extremely small and the interference is not determined with de Broglie phase but this small phase. This leads an unusual neutrino interference of the present work. The next singular term is from $1/\lambda$ in $\Delta_{\pi,l}$, and becomes $J_{\delta(\lambda)} / \sqrt{\pi\sigma_\nu |\vec{p}_\nu|^2}$, which is much smaller than $J_{\delta(\lambda)}$ in the present parameter region. The magnitude is inversely proportional to $|\delta t|$ and is independent from the \tilde{m}^2 for the general form of wave packet also.

Integrating t_1 and t_2 in a finite $T = T_\nu - T_\pi$, we have a slowly decreasing term $\tilde{g}(T, \omega_\nu)$ and a normal term G_0 . $\tilde{g}(T, \omega_\nu)$ is generated from the light-cone singularity and related term. $\tilde{g}(T, \omega_\nu) = g(T, \omega_\nu) - g(\infty, \omega_\nu)$ where

$$Tg(T, \omega_\nu) = -i \int_0^T dt_1 dt_2 \frac{\epsilon(\delta t)}{|\delta t|} e^{i\omega_\nu \delta t}, \quad (15)$$

and $\tilde{g}(\infty, \omega_\nu) = 0$. The normal term, TG_0 , is from the rest. Due to the rapid oscillation in δt , G_0 gets contribution from the microscopic δt region and is constant

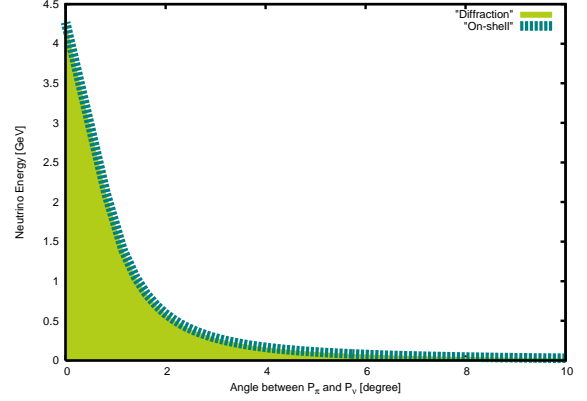


Figure 1: The angle between p_π and p_ν dependence of the neutrino energy is given. The horizontal axis shows the angle and the vertical axis shows the neutrino energy at $E_\pi = 10$ [GeV]. The normal term has a value along the boundary and the diffraction term has a value in a broad area below the normal term.

in a macroscopic T . This term does not depend on σ_ν and agrees with the normal probability obtained with the standard method of using plane waves. In the region $2p_\pi \cdot p_\nu > \tilde{m}$, $\Delta_{\pi,l}(\delta x)$ does not have the light-cone singularity and the diffraction term exists only in the kinematical region $2p_\pi \cdot p_\nu \leq \tilde{m}$.

We compute a total probability next. From an integration of neutrino's coordinates \vec{X}_ν , the total volume V is obtained and cancelled with the normalization of the initial pion state. The total probability, then, becomes a sum of the normal term G_0 and the diffraction term $\tilde{g}(T, \omega_\nu)$,

$$P = N_3 \int \frac{d^3 p_\nu}{(2\pi)^3} \frac{p_\pi \cdot p_\nu (m_\pi^2 - 2p_\pi \cdot p_\nu)}{E_\nu} [\tilde{g}(T, \omega_\nu) + G_0], \quad (16)$$

where $N_3 = 8Tg^2\sigma_\nu$ and $L = cT$ is the length of a decay region. At $T \rightarrow \infty$, the diffraction term vanishes and the probability P agrees with the value of the standard calculation of plane waves. At a finite T , the probability has the diffraction component, $\tilde{g}(T, \omega_\nu)$, which is stable under variation of the pion's energy.

Now we study each term of Eq. (16). In the normal term, G_0 , the energy and momentum are conserved well, and G_0 has a sharp peak at $p_\pi \cdot p_\nu = \tilde{m}^2/2$. Hence the factor $m_\pi^2 - 2p_\pi \cdot p_\nu$ in Eq. (16) becomes m_l^2 and a cosine of angle between \vec{p}_π and \vec{p}_ν is determined uniquely by the relation,

$$\cos \theta_{\pi,\nu} = \frac{(m_l^2 - m_\pi^2 + 2E_\pi E_\nu)}{2|\vec{p}_\pi||\vec{p}_\nu|}, \quad (17)$$

and the probability is proportional to m_l^2 . Integration of the neutrino's angle leads this integral independent of

the angle width, as far as it include the narrow peak. The value is independent also of σ_ν , which is consistent with the condition for the stationary state [15].

The diffraction component, $\tilde{g}(T, \omega_\nu)$, on the other hand, is present in a kinematical region, $|\vec{p}_\nu|(E_\pi - |\vec{p}_\pi|) \leq p_\pi \cdot p_\nu \leq \tilde{m}^2/2$. The energy and momentum are not exactly conserved in the space-time dependent amplitude Eq. (10). Hence $m_\pi^2 - 2p_\pi \cdot p_\nu$ in Eq. (16) is larger than m_l^2 , and the cosine of angle between \vec{p}_π and \vec{p}_ν is not uniquely determined. The dependence of the diffraction term upon the angle between \vec{p}_π and \vec{p}_ν is presented in Fig. 1 for $E_\pi = 10$ [GeV]. The angle, which is determined uniquely from the neutrino energy for the small wave packet, is not unique and is widely spread. From this behavior of the diffraction component, it might have been possible to reject the diffraction component and to take only the normal component in narrow band beam experiments if the neutrino's interaction position were observed. Comparison of the total cross section of satisfying this constraint with that of non-constraint events might have given direct signals of the diffraction component.

The diffraction term is slowly varying with both the distance and energy. The typical length L_0 of this universal behavior is L_0 [m] = $2E_\nu \hbar c / m_\nu^2 = 400 \times E_\nu [\text{GeV}] / m_\nu^2 [\text{eV}^2/c^4]$. Its magnitude was found to be about 10-20 % of the normal term in a suitable experimental situation and depends on geometry of the detector. In this paper we analyze total neutrino nucleon cross section, which is connected with the neutrino flux and its high energy behavior is understood well from a standard quark parton model.

5. Charged leptons

A probability of observing a neutrino or a charged lepton in the pion decay at a position of a macroscopic distance is written in the form [6, 7],

$$P = P_{\text{normal}} + P_{\text{diff}}^l. \quad (18)$$

In Eq. (18), P_{normal} is the normal term that is obtained from the decay probability G_0 in Eq. (16) and P_{diff}^l is the diffraction term that is determined from \tilde{g} in Eq. (16). The latter has not been included in calculating the total cross sections before and its effect is estimated here.

The diffraction term at a time T is described by its mass and energy in the universal form

$$P_{\text{diff}}^l = C_{\text{diff}} T \tilde{g}(T, \omega_l), \quad (19)$$

where C_{diff} is a constant and is obtained later. ω_l are small in neutrinos and large in charged leptons. $\tilde{g}(T, \omega_l)$

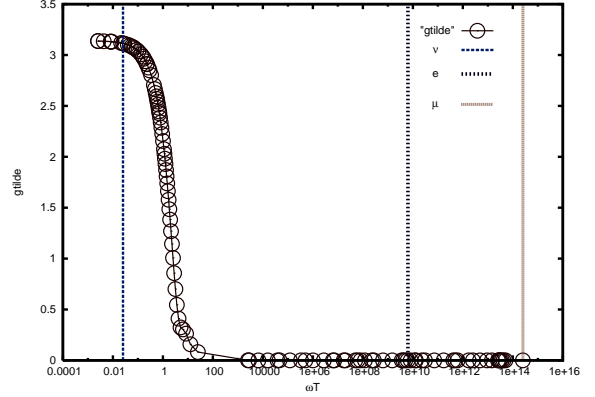


Figure 2: Values of $\tilde{g}(T, \omega)$ for ν , e and μ at $L = 10$ [m], $E = 1$ [GeV] and $m_\nu c^2 = 1$ [eV], $m_e c^2 = 0.5$ [MeV] and $m_\mu c^2 = 100$ [MeV].

is positive definite and decreases slowly with a distance $L = cT$, and vanishes at the infinite distance with the length scale $2cE_l/m_l^2$. This scale is macroscopic for neutrinos but is 10^{-10} [m] or less for the electron and muon. The magnitude of $\tilde{g}(T, \omega_l)$ is given in Fig. (2). At $L = 10$ [m], $E = 1$ [GeV] for the mass 1 [eV/ c^2] (ν), 0.5 [MeV/ c^2] (e) and 100 [MeV/ c^2] (μ), the values are,

$$\begin{aligned} \tilde{g}(T, \omega_\nu) &\approx 3, \\ \tilde{g}(T, \omega_e) &\approx 0, \\ \tilde{g}(T, \omega_\mu) &\approx 0. \end{aligned} \quad (20)$$

Hence the diffraction component at a macroscopic distance is finite in neutrinos and vanishes in others. It is striking that the neutrino flux has an additional term and is not equivalent to that of the charged lepton even though they are produced in the same decay process. This diffraction term is generated by the tiny neutrino mass and the interference term generated with the superposed waves of forming the light-cone singularity. Because P_{diff} is the interference term, it has unusual properties different from those of P_{normal} in the neutrino flavour and the energy and momentum. The neutrino diffraction furthermore is sensitive to the absolute neutrino mass. We study implications of the diffraction term to total cross sections, next.

6. Total cross sections of ν_μ -N scattering

A total cross section of a neutrino nucleon scattering is written in the form,

$$\sigma^\nu = \frac{M_N E_\nu G_F^2}{\pi} (Q + \bar{Q}/3), \quad (21)$$

using integrals of quark-parton distribution functions $q(x)$ and $\bar{q}(x)$, $Q = \int_0^1 dx x q(x)$, $\bar{Q} = \int_0^1 dx x \bar{q}(x)$.

The cross section is proportional to the neutrino energy and a current value is $\sigma_\nu/E = (0.677 \pm 0.014) \times 10^{-38} [\text{cm}^2/\text{GeV}]$ [3]. So experiments seem consistent with Eq. (21). However recent experiments of NOMAD [4] and MINOS [5] gave the total cross sections in wide energy ranges with small uncertainties and showed that the cross sections have slight energy dependences. They are compared with our theoretical calculations in the following.

The diffraction term was identified only recently. Here we include the diffraction term into the neutrino flux. The neutrino flux in pion decays is given as a sum of the normal and diffraction terms in the form

$$f = f_{\text{normal}}(1 + r_{\text{diff}}), \quad (22)$$

where r_{diff} is a ratio of the diffraction component over the normal component, and is a function of $\zeta = m_\nu^2 L / 2cE_\nu$,

$$r_{\text{diff}} = d_0 \tilde{g}(\zeta), \quad (23)$$

where the coefficient d_0 is determined from geometry.

When the detector is located at an end of a decay volume, the correction factor Eq. (23) is used. In an actual experiment, the detector is located in a distant region from a decay volume. There is soil between them and pions are stopped in a beam dump. The neutrino diffraction does not occur and neutrino propagates freely in this region. Since the wave packets of one σ_ν form a complete set, the wave packet size at the decay volume is the σ_ν determined with the detector. The neutrino flux at the end of the decay volume is computed with the diffraction term of the decay volume's length L and the wave packet size of the detector. Wave packets of this σ_ν propagate freely from the end of a decay volume to the detector. The final value of the neutrino flux at the detector is found using the factor Eq. (23). When a neutrino changes flavour in this period, the final probability for each flavour is written with a standard formula of flavour oscillation.

A true value of the total cross section, $\sigma(E)^{\text{true}}$, obtained with the total flux is connected with a cross section, $\sigma(E)^{\text{exp}}$, obtained with only the normal component of the flux by the ratio

$$\sigma(E)^{\text{true}} = \sigma(E)^{\text{exp}} \frac{1}{1 + r_{\text{diff}}}.$$

Conversely the experimental cross section is written as

$$\sigma(E)^{\text{exp}}/E = (1 + r_{\text{diff}})(\sigma(E)^{\text{true}}/E). \quad (24)$$

$\sigma(E)^{\text{true}}/E$ is believed a constant so the E-dependence of $\sigma(E)^{\text{exp}}/E$ is due to the E-dependence of r_{diff} , Eq. (23).

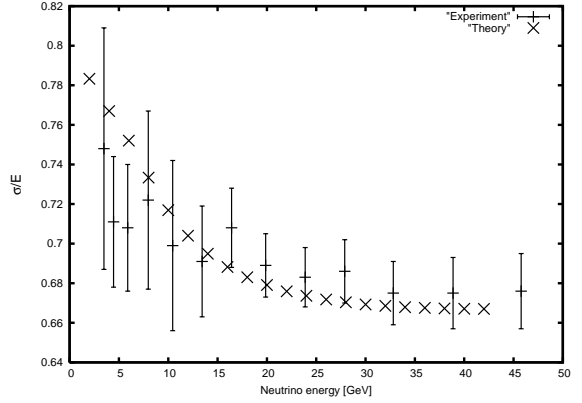


Figure 3: Total cross section of MINOS is compared with the sum of normal and diffraction terms. $m_\nu c^2 = 0.2 \text{ eV}$

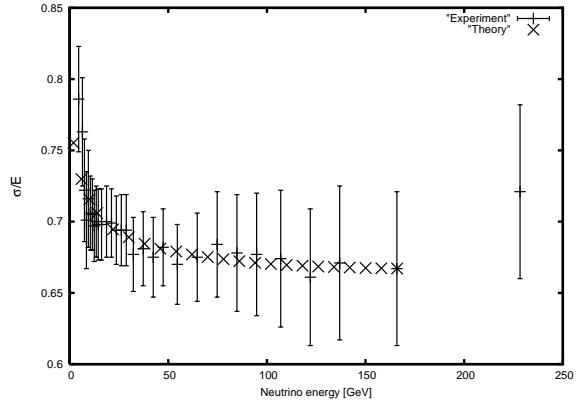


Figure 4: Total cross section of NOMAD is compared with the sum of normal and diffraction terms. $m_\nu c^2 = 0.2 \text{ eV}$

Since the diffraction term has different properties from those of the normal term, the corrections r_{diff} depends also on the geometry of the experiment and the material of the detector. In this paper we compute the cross sections using the experimental conditions of MINOS and NOMAD, which have presented precise cross sections of various energy ranges under the same condition, and compare them with the experiments.

The geometry of MINOS and NOMAD are the following. The lengths between the pion source and the neutrino detector, $L_{\text{det-so}}$, and those of the decay regions, $L_{\text{dec-reg}}$, are:

NOMAD : $L_{\text{det-so}} = 835 [\text{m}]$, $L_{\text{dec-reg}} = 290 [\text{m}]$,

MINOS : $L_{\text{det-so}} = 1040 [\text{m}]$, $L_{\text{dec-reg}} = 675 [\text{m}]$.

The detector size is $7 \times 3 \times 3 [\text{m}^3]$ for both experiments.

The wave packet size is estimated using the size of target nucleus. From the size of the nucleus of the mass number A , we have $\sigma_\nu = A^{2/3}/m_\pi^2$. For various material

the value are

$$\begin{aligned}\sigma_\nu &= 5.2/m_\pi^2; \text{ }^{12}\text{C nucleus}, \\ \sigma_\nu &= 14.3/m_\pi^2; \text{ }^{54}\text{Fe nucleus}.\end{aligned}\quad (25)$$

The total cross sections of MINOS and NOMAD experiments are compared with the theoretical values in Figs. (3) and (4). In theoretical calculations, effects of a pion beam spreading is included by taking an average of initial pion's angle from 0 to 10 [mrad] for NOMAD and from 0 to 15 [mrad] for MINOS. An effect due to a finite size of the initial pion was also estimated. We found that an plane wave approximation which was employed in this paper was very good.

The cross sections decrease quite slowly with the neutrino energy, which may be difficult to understand with the standard theory. The theoretical cross sections obtained by including the diffraction component into the neutrino flux showed the same behavior and agreed well with the experiments. Two experiments are actually different in the neutrino energy and geometry, but agreed with the theory. So the large cross sections at low energy regions may be attributed to the diffraction component.

We have compared only NOMAD and MINOS here. Many experiments are listed in particle data [3] and most of them have similar energy dependences and agree qualitatively with the diffraction's presence. It is important to notice that the magnitude of diffraction component is sensitive to geometry and if a kinematical constraint Eq. (17) on the angle between \vec{p}_π and \vec{p}_ν was required, only the events of the normal term was selected. Then the cross section should agree with that of the normal term.

7. Summary and implications

We showed that due to the diffraction component, P_{diff} , the neutrino flux was modified. The total neutrino cross sections of NOMAD and MINOS agreed with the theoretical calculations obtained using the modified neutrino flux. Thus the existence of the neutrino diffraction is consistent with experimental observations of the total cross sections. Although the diffraction component is determined by the average neutrino mass, its sensitivity of the existing data is not sufficient to find an information on the neutrino mass below 0.3 [eV/ c^2].

Other channels of interests are quasi-elastic or one pion production processes. The cross sections for $\nu + n \rightarrow \mu^- + p^+ (+\pi^0)$, $\nu + p^+ \rightarrow \mu^- + p^+ + \pi^+$ and $\bar{\nu} + p^+ \rightarrow \mu^+ + n (+\pi^0)$ and the neutral current process are known well theoretically using models such as

CVC, PCAC, and vector dominances and others. Recent experiments [19] showed that the cross sections have excesses of 20 – 50% and are consistent with the diffraction terms. Especially a proton is known to have large wave packet size in matter due to its small mass and the diffraction due to a proton target is enhanced and gives a finite contribution despite its small energy ratio, $m_{proton}/m_{nucleus}$.

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